

Foundation Design (3rd Edition)

Chapter 2, Problem 11QPP

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Problem

For the footing in Example, compute the factor of safety required for a probability of failure of 5 × 10^{−4} assuming the COV of the demand is 0.15

Example

Assume we are building a foundation on a saturated clay deposit with an average undrained shear strength of 2,000 lb/ft² and a COV of 0.2. The average unit weight for the soil is 102 lb/ft³ with a COV of 0.05. What is the mean shortterm nominal bearing capacity of a continuous footing founded at a depth of 5 ft? What is the probability that the nominal bearing capacity of a certain footing would be less than 75 percent of the mean nominal bearing capacity if the undrained shear strength and unit weight are normally distributed and independent?

Solution

Using Equation 1, we compute the mean nominal bearing capacity as

$$\mu_{b_o} = 5.7(2,000) + 5(102) = 11,910 \text{ lb/ft}^2$$

To compute the standard deviation of nominal bearing capacity, we must first compute the standard deviations of the undrained shear strength and unit weight from their COVs using Equation 2:

$$\begin{aligned} \sigma_{s_u} &= 0.2(2,000) \\ &= 400 \text{ lb/ft}^2 \\ \sigma_{\gamma} &= 0.05(102) \\ &= 5.1 \text{ lb/ft}^3 \end{aligned}$$

Using Equation 3, we compute the standard deviation of the nominal bearing capacity as

$$\sigma_{b_o} = \sqrt{5.7^2(400)^2 + 5^2(5.1)^2} = 2,280 \text{ lb/ft}^2$$

We can then compute the probability that the nominal bearing capacity is less than 75 percent of the mean value (0.75 × 11,910 = 8,933) using Equation 4:

$$\begin{aligned} P(q \leq 8,933) &= \Phi\left(\frac{8,933 - 11,910}{2,280}\right) \\ &= \Phi(-1.31) \\ &= 1 - \Phi(1.31) \end{aligned}$$

Using Table,

$$P(q \leq 8,933) = 1 - 0.90490 = 0.0951 \text{ or } 9.51\%$$

Equation 1

$$\mu_c = a\mu_X + b\mu_Y$$

Equation 2

$$COV = \frac{\sigma_X}{\mu_X}$$

Equation 3

$$\sigma_c = \sqrt{a^2\sigma_X^2 + b^2\sigma_Y^2}$$

Equation 4

$$P(X \leq a) = \Phi\left(\frac{a - \mu_X}{\sigma_X}\right)$$

TABLE B1 CUMULATIVE STANDARD NORMAL DISTRIBUTION PROBABILITY.

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$$

<i>x</i>	$\Phi(x)$	<i>x</i>	$\Phi(x)$	<i>x</i>	$\Phi(x)$	<i>x</i>	$\Phi(x)$
0.00	0.500000000	0.14	0.55566005	0.28	0.610321248	0.42	0.662752723
0.01	0.503989356	0.15	0.559617602	0.29	0.614091881	0.43	0.666402179
0.02	0.507978314	0.16	0.563559463	0.30	0.617911422	0.44	0.670051446
0.03	0.511966473	0.17	0.567494932	0.31	0.621719522	0.45	0.673644780
0.04	0.515953437	0.18	0.571423716	0.32	0.625515835	0.46	0.677241890
0.05	0.519938806	0.19	0.575345435	0.33	0.629300019	0.47	0.680822491
0.06	0.523922183	0.20	0.579258769	0.34	0.633071736	0.48	0.684386303
0.07	0.527903170	0.21	0.583166163	0.35	0.636830651	0.49	0.687933951
0.08	0.531881372	0.22	0.587064423	0.36	0.640576433	0.50	0.691462461
0.09	0.535856393	0.23	0.590954115	0.37	0.644308755	0.51	0.694974269
0.10	0.539827837	0.24	0.594834872	0.38	0.648027292	0.52	0.698468212
0.11	0.543796313	0.25	0.598706766	0.39	0.651731727	0.53	0.701944075
0.12	0.547758426	0.26	0.60258113	0.40	0.655421742	0.54	0.705401484
0.13	0.551716787	0.27	0.606449873	0.41	0.659097026	0.55	0.708848313

<i>x</i>	$\Phi(x)$	<i>x</i>	$\Phi(x)$	<i>x</i>	$\Phi(x)$	<i>x</i>	$\Phi(x)$
0.56	0.712260281	0.88	0.810570345	1.20	0.884930330	1.52	0.935744512
0.57	0.715661151	0.89	0.813226707	1.21	0.886860554	1.53	0.936991406
0.58	0.719042091	0.90	0.815939875	1.22	0.888767563	1.54	0.938219823
0.59	0.722404675	0.91	0.818588745	1.23	0.890651448	1.55	0.939429242
0.60	0.725746882	0.92	0.821213520	1.24	0.892512303	1.56	0.940620059
0.61	0.729069006	0.93	0.823814458	1.25	0.894350226	1.57	0.941792444
0.62	0.732371107	0.94	0.826391220	1.26	0.896165119	1.58	0.942946023
0.63	0.735651708	0.95	0.828943874	1.27	0.897957585	1.59	0.944082597
0.64	0.738913700	0.96	0.831472393	1.28	0.899727432	1.60	0.945200708
0.65	0.742155889	0.97	0.833976754	1.29	0.901474671	1.61	0.946301072
0.66	0.745378085	0.98	0.836456941	1.30	0.903119915	1.62	0.947383382
0.67	0.748571105	0.99	0.838912940	1.31	0.904692082	1.63	0.948449252
0.68	0.751741770	1.00	0.841447466	1.32	0.906292891	1.64	0.949497417
0.69	0.754890296	1.01	0.843975255	1.33	0.907924064	1.65	0.950528532
0.70	0.758006348	1.02	0.846435770	1.34	0.909487728	1.66	0.951542314
0.71	0.761147932	1.03	0.848949977	1.35	0.910987209	1.67	0.952540178
0.72	0.764237003	1.04	0.851430050	1.36	0.912463038	1.68	0.953523142
0.73	0.767263498	1.05	0.853884944	1.37	0.913916549	1.69	0.954494023
0.74	0.770259003	1.06	0.856327700	1.38	0.915346967	1.70	0.955445357
0.75	0.773232648	1.07	0.858760346	1.39	0.917735561	1.71	0.956367063
0.76	0.776172708	1.08	0.859928910	1.40	0.919243341	1.72	0.957263779
0.77	0.779135004	1.09	0.862143428	1.41	0.920730159	1.73	0.958148462
0.78	0.782043662	1.10	0.864313939	1.42	0.922196159	1.74	0.959019491
0.79	0.784914116	1.11	0.866450447	1.43	0.923641490	1.75	0.959880843
0.80	0.787844601	1.12	0.868643119	1.44	0.925066300	1.76	0.960766697
0.81	0.791029912	1.13	0.870761888	1.45	0.926470740	1.77	0.961656430
0.82	0.793931946	1.14	0.872856849	1.46	0.927854963	1.78	0.962540200
0.83	0.796797908	1.15	0.874928064	1.47	0.929219123	1.79	0.963273044
0.84	0.799634807	1.16	0.876975997	1.48	0.930563077	1.80	0.964004981
0.85	0.802337437	1.17	0.878999516	1.49	0.931887882	1.81	0.9646852106
0.86	0.805105479	1.18	0.880999893	1.50	0.933192799	1.82	0.9653620498
0.87	0.807849798	1.19	0.882976804	1.51	0.934478288	1.83	0.9660375031

<i>x</i>	$\Phi(x)$	<i>x</i>	$\Phi(x)$	<i>x</i>	$\Phi(x)$	<i>x</i>	$\Phi(x)$
1.84	0.967115881	2.16	0.984613665	2.48	0.993430881	2.80	0.997444870
1.85	0.967843225	2.17	0.9849496377	2.49	0.993612845	2.81	0.997522925
1.86	0.968565227	2.18	0.985131269	2.50	0.993790335	2.82	0.997599818
1.87	0.969285091	2.19	0.985317882	2.51	0.993963442	2.83	0.997675800
1.88	0.969949961	2.20	0.985509652	2.52	0.994132258	2.84	0.997744323
1.89	0.970621020	2.21	0.985647419	2.53	0.994296874	2.85	0.997814039
1.90	0.971283440	2.22	0.985760616	2.54	0.994457377	2.86	0.997881795
1.91	0.971933393	2.23	0.9858712679	2.55	0.994613854	2.87	0.997947641
1.92	0.972571650	2.24	0.985974459	2.56	0.994766202	2.88	0.998011624
1.93	0.973196581	2.25	0.9860775527	2.57	0.994915074	2.89	0.998073791
1.94	0.973810155	2.26	0.9861809375	2.58	0.995059984	2.90	0.998134187
1.95	0.974411940	2.27	0.9862836208	2.59	0.995200203	2.91	0.998192856
1.96	0.975002105	2.28	0.986386136	2.60	0.995338812	2.92	0.998249843
1.97	0.975586815	2.29	0.9864891742	2.61	0.995472889	2.93	0.998305190
1.98	0.976164236	2.30	0.986592580	2.62	0.995603512	2.94	0.998358939
1.99	0.976734552	2.31	0.9866955923	2.63	0.995730757	2.95	0.998411130
2.00	0.977298968	2.32	0.986792561	2.64	0.995854699	2.96	0.998461805
2.01	0.977844406	2.33	0.9868946924	2.65	0.995975411	2.97	0.998511001
2.02	0.978383006	2.34	0.986993130	2.66	0.996092967	2.98	0.998558758
2.03	0.978916217	2.35	0.987087459	2.67	0.996207328	2.99	0.998604113
2.04	0.979434837	2.36	0.9871806252	2.68	0.996318892	3.00	0.998650162
2.05	0.979937785	2.37	0.991169597	2.69	0.996427399	3.01	0.998697502
2.06	0.980000730	2.38	0.991343681	2.70	0.996533026	3.02	0.998736127
2.07	0.9800773828	2.39	0.991515814	2.71	0.996635840	3.03	0.998777231
2.08	0.9801437234	2.40	0.991681848	2.72	0.996736094	3.04	0.998817109
2.09	0.9802101100	2.41	0.991820240	2.73	0.996834384	3.05	0.998855793
2.10	0.981355579	2.42	0.992239746	2.74	0.996928041	3.06	0.998893315
2.11	0.982570822	2.43	0.992450589	2.75	0.997020237	3.07	0.998929706
2.12	0.982969077	2.44	0.992656369	2.76	0.997109932	3.08	0.998964997
2.13	0.983414193	2.45	0.992857189	2.77	0.997197185	3.09	0.998999218
2.14	0.983822617	2.46	0.993031149	2.78	0.997282855	3.10	0.999032397
2.15	0.984222293	2.47	0.9931944347	2.79	0.997364598	3.11	0.999064563

3.12	0.999595745	3.44	0.999709143	3.76	0.999915043	4.8	0.999999542
3.13	0.99912968	3.45	0.999719707	3.77	0.999915376	4.9	0.999999707
3.14	0.999155261	3.46	0.999729912	3.78	0.999915686	5.0	0.999999870
3.15	0.999181368	3.47	0.999739771	3.79	0.999915976	5.1	0.999999945
3.16	0.999211154	3.48	0.999749293	3.80	0.999916256	5.2	0.999999988
3.17	0.999237805	3.49	0.999758490	3.81	0.999916517	5.3	0.999999998
3.18	0.999262625	3.50	0.999767371	3.82	0.999916764	5.4	0.999999999
3.19	0.999286636	3.51	0.999775947	3.83	0.999916998	5.5	0.999999999
3.20	0.999311282	3.52	0.999784227	3.84	0.999917221	5.6	0.999999999
3.21	0.999336325	3.53	0.999792220	3.85	0.999917434	5.7	0.999999999
3.22	0.999359047	3.54	0.999799936	3.86	0.999917636	5.8	0.999999999
3.23	0.999381040	3.55	0.999807384	3.87	0.999917828	5.9	0.999999999
3.24	0.999402352	3.56	0.999814573	3.88	0.999917999	6.0	0.999999999
3.25	0.999422975	3.57	0.999821509	3.89	0.999918148	6.1	0.999999999
3.26	0.999442939	3.58	0.999828203	3.90	0.999918300	6.2	0.999999999
3.27	0.999462902	3.59	0.999834691	3.91	0.999918446	6.3	0.999999999
3.28	0.999482865	3.60	0.999840891	3.92	0.999918586	6.4	0.999999999
3.29	0.999502830	3.61	0.999846901	3.93	0.999918725	6.5	0.999999999
3.30	0.999516736	3.62	0.999852808	3.94	0.999918859	6.6	0.999999999
3.31	0.999530642	3.63	0.999858536	3.95	0.999918987	6.7	0.999999999
3.32	0.999544513	3.64	0.999864081	3.96	0.999919112	6.8	0.999999999
3.33	0.999557770	3.65	0.999869480	3.97	0.999919234	6.9	0.999999999
3.34	0.999571108	3.66	0.999874732	3.98	0.999919352	7.0	0.999999999
3.35	0.999585462	3.67	0.999879825	3.99	0.999919468	7.1	0.999999999
3.36	0.999599839	3.68	0.999884776	4.00	0.999919579	7.2	0.999999999
3.37	0.999614319	3.69	0.999889573	4.1	0.999919675	7.3	0.999999999
3.38	0.999628879	3.70	0.999894220	4.2	0.999919765	7.4	0.999999999
3.39	0.999643537	3.71	0.999898630	4.3	0.999919846	7.5	0.999999999
3.40	0.999658301	3.72	0.999902909	4.4	0.999919916	7.6	0.999999999
3.41	0.999673179	3.73	0.999907060	4.5	0.999919976	7.7	0.999999999
3.42	0.999688124	3.74	0.999911090	4.6	0.999919999	7.8	0.999999999
3.43	0.999698239	3.75	0.999915183	4.7	0.999919966	7.9	0.999999999

Step 2 of 4

Calculate the standard deviation of the safety margin as a function of standard deviation of the demand.

$$\sigma_s = \sqrt{\sigma_c^2 + \sigma_D^2}$$

From the example 2.2 the value of the standard deviation is 2280 lb/ft².

Here, $\sigma_D = COV_D \mu_D$.

Substitute 2280 lb/ft² for σ_c and 0.15 for COV_D .

$$\sigma_s = \sqrt{2280^2 + 0.15^2 \mu_D^2}$$

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Step 3 of 4

Calculate the value of μ_D by using the following formula:

$$P = \Phi\left(\frac{\mu_D - \mu_s}{\sigma_s}\right)$$

Here, the probability of failure is P .

Substitute 5×10^{-4} for P , 11910 - μ_D for μ_s and $\sqrt{2280^2 + 0.15^2 \mu_D^2}$ for σ_s .

$$\begin{aligned} 5 \times 10^{-4} &= \Phi\left(\frac{\mu_D - (11910 - \mu_D)}{\sqrt{2280^2 + 0.15^2 \mu_D^2}}\right) \\ &= \Phi\left(\frac{2\mu_D - 11910}{\sqrt{2280^2 + 0.15^2 \mu_D^2}}\right) \end{aligned}$$

Solve the equation to get μ_D .

$$\mu_D = 4135$$

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Step 4 of 4

Calculate the factor safety as follows:

$$F = \frac{\mu_c}{\mu_D}$$

Substitute 11910 lb/ft² for μ_c and 4135 for μ_D .

$$\begin{aligned} F &= \frac{11910}{4135} \\ &= 2.88 \\ &\approx 2.9 \end{aligned}$$

Therefore, the value of factor of safety is **2.9**.

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